

RADIATION IN A LAYER OF NONGRAY GAS BETWEEN GRAY WALLS

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Radiative heat transfer formulas are derived for an isothermal layer of gas and gray walls; they are compared with previously published formulas. The method of derivation makes it possible to refine the formulas obtained.

Engineering methods of calculating heat transfer in furnaces and combustion chambers with a nongray gas are characterized by the averaging of the gas temperature; the gas volume figures as a single isothermal zone. We examine the calculations for the case of a nonturbid gas between gray walls. The literature includes several different formulas that take the multiple reflections of the fluxes into account [1-6]; another formula was recently proposed in [7]. All these formulas are characterized by a serious error (up to tens of percent), and the problem has still not been satisfactorily solved. We have derived general formulas that are more accurate than those previously employed.

A plane-parallel layer of gas between gray walls is a system optimal in simplicity and generality and most convenient for a comparison of different methods. The fact that the emissivity ϵ and the quantities derived from it (D and ϵ_*) depend on the path of the ray and, hence, on the multiplicity of reflections, places limitations on the methods of determinants or matrix algebra. The method of multiple reflections is more convenient.

In the calculations, we employ the characteristic of the total spectrum of the gas ϵ_* . Let $K(x)$ be the mean absorption coefficient on the path x for the self-radiation of the gas. Then the transmissivity on the interval x is equal to $D = \exp(-Kx)$ and $\epsilon_* = \epsilon(1 - D)^{-1}$. As $x \rightarrow \infty$, but with preservation of the value of K referred to a certain interval x_1 , we obtain $\epsilon_*(x_1) \rightarrow \epsilon(\infty)$, which also defines the physical significance of ϵ_* . The quantity ϵ_* is determined only by the absorption spectrum of the gas on the interval x_1 . The blackbody radiation density fraction $1 - \epsilon_*$ corresponds to the spectral window. The function $\epsilon_*(x)$ is an increasing one. As $x \rightarrow 0$, $\epsilon_* \rightarrow \alpha/K_0$, where K_0 is the maximum absorption coefficient. This value of ϵ_* is minimum. As $x \rightarrow \infty$, $\epsilon \rightarrow \epsilon_\infty$, and, hence, $\epsilon_* \rightarrow \epsilon_\infty$. It is a widely held opinion that the quantity ϵ_∞ is much less than unity. A number of sources give functions $\epsilon_\infty(T)$. However, it is known [8, 9] that as $x \rightarrow \infty$ we get absorption by the far wings of the strong rotational lines, i. e., absorption in the spectral windows. In atmospheric optics an absorption coefficient for the spectral windows is introduced [9]. Then as $x \rightarrow \infty$ the quantity ϵ becomes very close to unity. Apparently, somewhere on the $\epsilon(x)$ -curve (near x_0) there is a marked change in the derivative. The values of ϵ_∞ given in the literature should be understood as the emissivity of the interval x_0 .

Although the quantities ϵ , D , and ϵ_* depend on the multiplicity of reflections, to curtail the series it is necessary to use certain mean values. In furnace and combustion chamber engineering the reflection coefficients usually have values less than 0.3-0.4. Under these conditions the greatest accuracy is required in computing the fluxes corresponding to the first and second reflections.

The calculation reduces to the computation of the dimensionless coefficients M_{mk} , which determine the resultant flux for the pair of zones m and k :

$$q_{pm} = \sigma (M_{km} T_k^4 - M_{mk} T_m^4).$$

The total resultant flux of zone m is equal to the sum of the q_{pm} over all the zones participating in heat transfer, including the zone m itself. Denoting the surfaces by the subscripts 1 and 2, and the gas by the subscript g , we find M_{g1} . When multiple reflections are taken into account we have ϵ_1 —the primary flux incident on surface 1 (at $\sigma T_g^4 = 1$). To this there is added the flux $\epsilon_1 R_2 D_2 / D_1$ reflected from surface 2. The total flux $\epsilon_1 (1 + R_2 D_2 / D_1)$ begins to undergo multiple reflection. Its first principal part has a transmission "there" D_2 / D_1 , and "back" D_3 / D_2 or a total transmission for the first cycle of reflections $(D_2 / D_1)(D_3 / D_2) = D_3 / D_1$.

The second part has a total transmission $(D_3 / D_2)(D_4 / D_3) = D_4 / D_2$. We assume that: a) the total transmission for

both parts is equal to D_3/D_1 as for the first cycle; and b) it is preserved in the subsequent reflection cycles. The sum of the geometric progression, multiplied by A_1 , gives the required coefficient

$$M_{g1} = A_1 \varepsilon_1 \frac{1 + R_2 D_2/D_1}{1 - R_1 R_2 D_2/D_1}. \quad (1)$$

In the presence of multiple reflection, it is primarily the spectral components with high absorption coefficients that disappear. This is manifested in the inequality $D_{n+1}/D_n > D_n/D_{n-1}$. Then, the multiplier in the denominator of Eq. (1) $-D_3/D_1$ is too low. The quantity M_{g1} is also too low, because the increase in emissivity with elongation of the rays in multiple reflection has not been taken into account. The coefficient M_{g2} is determined by symmetry, and M_{gg} is obtained from the flux balance

$$M_{gg} = 2\varepsilon_1 - M_{g1} - M_{g2} = \varepsilon_1 \frac{(R_1 + R_2)(1 - D_2/D_1) + 2R_1 R_2 (D_2 - D_3)/D_1}{1 - R_1 R_2 D_2/D_1}.$$

The quantity $2R_1 R_2 (D_2 - D_3)/D_1$ can be discarded to simplify the formula, and also because the values of M_{g1} and M_{g2} are too low. Then there is a discrepancy in the balance. The expression for M_{1g} is the same as that for M_{g1} but with a_{g1} substituted for ε_1 , as recommended by Hottel [3]. In this case, the quantities D_1 and ε_{*1} are calculated from a_{g1} and are denoted by a prime:

$$D'_i = (\alpha')^{-1} (da_{g1}/dx)_{x=x_i}, \quad \alpha' = (da_{g1}/dx)_{x=0} = \alpha(T_1), \quad \varepsilon'_{*1} = a_{g1}/(1 - D'_1) \text{ etc.}$$

In determining M_{12} it is necessary to take multiple reflection in the spectral window and in the spectrum of the gas separately into account. Reasoning similar to that above gives

$$M_{12} = A_1 A_2 (1 - \varepsilon'_{*1})(1 - R_1 R_2)^{-1} + \varepsilon'_{*1} A_1 A_2 D'_1 (1 - R_1 R_2 D'_3/D'_1)^{-1}. \quad (2)$$

The quantity M_{11} is determined from the flux balance

$$M_{11} = A_1 - M_{1g} - M_{12} = A_1^2 \frac{R_2(1 - \varepsilon'_{*1})}{1 - R_1 R_2} + \varepsilon'_{*1} A_1 R_2 \frac{D'_1 + D'_2 - (R_1 D'_3 + D'_2)/D'_1}{1 - R_1 R_2 D'_3/D'_1}.$$

The use of the approximate formulas obtained encounters an obstacle in that the accuracy of the known functions $D(x, T)$ does not satisfy the calculation requirements. To compute D_1 and ε_{*1} (in our notation) Hottel recommends the rough formulas [3]

$$D_1 = \frac{\varepsilon_2}{\varepsilon_1} - 1, \quad \varepsilon_{*1} = \frac{\varepsilon_1^2}{2\varepsilon_1 - \varepsilon_2}.$$

If the reflection coefficients of these surfaces are high, it is recommended to use quantities D_i and ε_{*i} with a larger subscript i . They are defined in terms of ε_1 and ε_3 or ε_1 and ε_4 . It is preferable to define D_i and ε_{*i} in terms of the derivatives of the function $\varepsilon(x)$, as in our notation. Nevskii [5] has given graphs of the function $D(T, x)$ for CO_2 and H_2O on the temperature interval $400-1200^\circ \text{C}$. Tables for CO_2 can be found in [10], which also gives a formula for $D(T, x)$ derived from Schack's formula [2] and valid on the intervals $0 \leq x \leq 0.4 \text{ m} \cdot \text{atm}$, $200 \leq t \leq 2000^\circ \text{C}$. The accuracy of all these data is unsatisfactory at large optical thicknesses. The error of the ratios D_i/D_j employed is even higher. The development of accurate functions $D(x, T)$ requires even more accurate functions $\varepsilon(x, T)$.

Comparison of the formulas derived. Monograph [3] gives more general heat-transfer formulas, from which our coefficients M_{1g} and M_{12} can be derived. For comparison, it is necessary to set $T_1 = T_g$ and then $a = \varepsilon$. Unfortunately, the denominators of the formulas of interest to us (4-104) and the first term of formula (4-105) are incorrect. The errata section is itself in error. A consistent derivation gives (in our notation):

$$M_{1g} = \varepsilon_1 A_1 (1 + D_1 R_2)(1 - D_1^2 R_1 R_2)^{-1}, \quad (3)$$

$$M_{12} = A_1 A_2 [(1 - \varepsilon_{*1})(1 - R_1 R_2)^{-1} + \varepsilon_{*1} D_1 (1 - D_1^2 R_1 R_2)^{-1}].$$

A comparison with (1) and (2) shows that here D_1 is used instead of D_2/D_1 and D_1^2 instead of D_3/D_1 , etc. Table 1 shows that the difference between the quantities compared is important. Hottel uses the method of determinants. The method of multiple reflections is more flexible. For example, instead of summation with approximate mean values of the transmissivities, it is possible to cut off the series obtained after the first terms. The error is small if R_1 and R_2 are small, and if x is sufficiently large.

In conclusion, we compare even simpler formulas for the heat transfer between a gas and boundary walls ($R_1 = R_2 = R$). Table 2 gives values of the coefficients M according to various sources.

$$M_1 = 0.5\epsilon_1(1 + A) \text{ acc. to [1],}$$

$$M_2 = \epsilon_1 A \left[1 - \left(1 - \frac{\epsilon_1}{\epsilon_\infty} \right) R \right]^{-1} \text{ acc. to [2],}$$

$$M_3 = 2\epsilon_1 A(1 + A)^{-1} \text{ acc. to [7],}$$

$$M_4 = \epsilon_1 A(1 - D_1 R)^{-1} \text{ acc. to Eq. (3),}$$

$$M_5 = \epsilon_1 A(1 - CR)^{-1} \text{ acc. to Eq. (1).}$$

The expressions for the quantities M_2 and M_4 are similar, if it is taken into account that $D = 1 - \epsilon_1/\epsilon_{*1}$. Obviously, ϵ_{*1} is insufficient, and ϵ_∞ is too large. Therefore, M_2 is everywhere too high, and M_4 is everywhere too low. In our formula for M_5 , $C = (D_3/D_1)^{1/2}$. The values of D_1 for small ϵ were estimated from the published data. For large ϵ this is not yet possible, and C is adjusted according to one of the values of M_0 . In this case we have a good interpolation formula for M_{1g} with respect to R . The quantities M_0 , M_2 and M_3 were taken from [7], where the M_0 are said to be the most accurate. We added the values of M_1 , M_4 and M_5 , taking the same values for $\epsilon(x')$ as in [7]. We determined values of $x'(\epsilon)$ from nomograms [5] and then calculated $D_1(x')$, $D_2(2x')$ and $D_3(3x')$. Here, we introduced an additional error in that the effective path length after the first reflection is somewhat less than $2x'$, after double reflection less than $3x'$, etc.

Table 1. Transmissivities on the Interval x in Carbon Dioxide According to the Data of [10]

x , m · atm	t , °C	D_1	D_2	D_3	D_2/D_1	$\sqrt{D_3/D_1}$
0.01	1400	0.196	0.128	0.103	0.654	0.72
0.01	400	0.109	0.0739	0.0553	0.677	0.71
0.1	1400	0.0453	0.0249	0.0159	0.55	0.59
0.1	400	0.0149	0.0052	0.0026	0.35	0.42

Given more accurate data it is possible directly to relate the quantities C and ϵ for each gas. A very rough approximation for all gases and $\epsilon \leq 0.4$ is

$$C = \frac{0.1 - \lg \epsilon}{2.1}$$

NOTATION

A_1 and A_2 are absorption coefficients for gray walls; $R_1 = 1 - A_1$; $R_2 = 1 - A_2$; $x_j = pl_j$ is the effective optical thickness, m · atm; p is the partial pressure of active component of gas, atm; l_j is the effective thickness, m; $x_i = \sum_j x_j$; $\epsilon_i = \epsilon(x_i)$ is the emissivity of a layer of thickness x_i ($i = 1, 2, \dots$); $D_i(x_i)$ is the transmissivity of a layer of thickness x_i ; $\epsilon_{*i} \equiv \epsilon_*(x_i)$ is the emissivity of a half-space with mean absorption coefficient the same as for a layer of thickness x_i ; M_{mk} is the mean probability of an energy quantum being emitted by zone m , entering zone k directly or indirectly and being absorbed by zone k ; σT^4 is the blackbody radiation density, W/m^2 ; q_p is the resultant flux density; α_{g1} is the absorption coefficient of a gas with respect to wall radiation; α' and D'_i are explained in the text; C is the mean transmissivity of a layer for "there" and "back" reflections; M_0, M_1, M_2, \dots are the values of M_{mk} for a closed homogeneous vessel and gas according to various sources;

REFERENCES

1. H. C. Hottel and R. B. Egbert, Trans. Am. Soc. Mech. Engrs., **53**, 297, 1941; Trans. Am. Inst. Chem. Engrs., **38**, 531, 1942.
2. A. Schack, Die industrielle Wärmeübergang, Düsseldorf, 1948.
3. W. H. McAdams, Heat Transmission [Russian translation], Metallurgizdat, 1961.
4. E. R. G. Eckert and R. M. Drake, Heat and Mass Transfer [Russian translation], Gosénergoizdat, 1961.
5. A. S. Nevskii, Radiative Heat Transfer in Metallurgical Furnaces and Boiler Combustion Chambers [in Russian], Metallurgizdat, 1958.

Table 2. Comparison of Coefficients M Characterizing the Heat Transfer Between a Gas and Boundary Walls From Various Published Sources

Gas	ϵ	A	M_0	M_1	M_2	M_3	$\sqrt{D_3/D_1}$	C	M_4	D_4	M_4
H ₂ O $x=0.01 \text{ m} \cdot \text{atm}$ $t=1200^\circ \text{ C}$	0.022	0.4	0.018	0.0154	0.021	0.0125	0.85	—	0.018	0.78	0.0165
		0.8	0.021	0.0198	0.022	0.0195			0.0212		0.0208
H ₂ O $x=0.1 \text{ m} \cdot \text{atm}$ $t=0^\circ \text{ C}$	0.305	0.2	0.084	0.183	0.129	0.1015	—	0.3	0.080	≈ 0	
		0.6	0.207	0.244	0.249	0.2285			0.208		
CO ₂ $x=0.01 \text{ m} \cdot \text{atm}$ $t=100^\circ \text{ C}$	0.038	0.2	0.013	0.023	0.023	0.0126	0.6	—	0.0146	≈ 0.1	0.0083
		0.4	0.021	0.027	0.031	0.0217			0.0238		0.0162
		0.6	0.027	0.030	0.034	0.0285			0.0300		0.0238
		0.8	0.033	0.034	0.036	0.0338				0.0345	0.0301
CO ₂ +H ₂ O $x_{\text{CO}_2}=0.08 \text{ m} \cdot \text{atm}$ $x_{\text{H}_2\text{O}}=0.08 \text{ m} \cdot \text{atm}$ $t=900^\circ \text{ C}$	0.275	0.2	0.085	0.165	0.130	0.0915	—	0.3	0.072	≈ 0	
		0.4	0.140	0.192	0.194	0.1570			0.134		
		0.6	0.190	0.220	0.231	0.2060			0.187		
		0.8	0.235	0.250	0.256	0.2445				0.234	

JOURNAL OF ENGINEERING PHYSICS

6. A. G. Blokh, Fundamentals of Radiative Heat Transfer [in Russian], Gosénergoizdat, 1962.
7. S. Splett, Brenstoff—Wärme—Kraft. , 17, no. 2, 70, 1965.
8. R. Goody, Atmospheric Radiation [Russian translation], Mir, 1966.
9. V. E. Zuev, Transparency of the Atmosphere for Visible and Infrared Rays [in Russian], Sovetskoe radio, 1966.
10. S. P. Detkov and V. N. Girs, Izvestiya VUZ. Chernaya metallurgiya, no. 2, 162, 1967.

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